CONTINUUM MODEL OF IMPACT EROSION

A. P. Trunev and V. M. Fomin

UDC 541.24:532.5

The process of erosional destruction of materials is analyzed within the framework of the model of a damaged continuum. Emphasis is given to nonsteady erosion typical of cavitational damage and damage caused by drops of liquid. The model predicts the loss of transparency of the material, the time of beginning of destruction, and the rate of erosion up to the establishment of a microrelief with depressions of constant depth on the eroded surface. It is shown that in the final stage the depth of wear increases in proportion to the logarithm of the time of the process. The proposed theory combines the concept of fatigue fracture advanced in [1] (also see the discussion of the time dependence of cavitational fractures in [2]) and the principles of the mechanics of a damaged continuum. Due to its great practical importance, the problem of modeling nonsteady erosion has attracted the attention of many investigators. Most of the existing erosion models were examined in [3, 4]. A large amount of experimental data on rainfall erosion of different materials was represented in the semiempirical model in [5].

1. General Formulation of the Problem. Impact erosion develops under conditions of two-phase flow about bodies. It is convenient to examine abstractions apart from a specific form of erosive medium, here assigning the necessary boundary conditions on the surface undergoing destruction. The problem of erosion can be examined as a problem of two-phase flow about a body [6, 7] and a problem of fracture mechanics [8]. Let us dwell on the latter case. The two-phase flow is a source of particles which create shock loads on the surface of the body (for the sake of definiteness, we will examine erosion by drops). The reaction of the material to the impact of these particles may differ, depending on the properties of the material and the parameters of the load. Nonsteady erosion is generally seen under low-intensity loads. We thus propose that the material is capable of withstanding a fairly large number of impacts on one contact spot before the thickness of the layer of material removed is comparable to the diameter of the drops (in fact, this number ranges within 10^3-10^7 impactsΔ ($\pi d_p^2/4$), where d_p is the diameter of a drop). In this case, erosion can be regarded as the result of continuous accumulation of damages in the material. To describe the state of the medium before fracture, we introduce a scalar measure of damage to the medium $\omega(x, y, z, t) \geq 0$ with normalization: $\omega = 1$ at the moment of complete fracture. The following damage summation law is valid:

$$d\omega/dt = \Phi(\omega, \sigma_{ik}, \{\xi_\omega\}), \tag{1.1}$$

where σ_{ik} is the stress tensor; $\{\xi_\omega\}$ is the set of parameters characterizing the damaged medium.

Let us examine the erosion of a material which has been damaged beforehand. After the elapse of a certain amount of time, the material in the surface layer will be completely destroyed, removal of material will begin, and motion will be imparted to the surface of the body. We will assume that the equation of the surface is $z + h(x, y, t) = 0$, while two-phase flow occupies the region $z + h(x, y, t) > 0$. The condition of the moving boundary has the form

$$\omega(x, y, -h(x, y, t), t) = 1. \tag{1.2}$$

When a material with a coating undergoes erosion, the fracture condition may be satisfied simultaneously on the surface and on the boundary between the coating and substrate. However, we will not examine this situation here. The duration of the latent period is determined as the solution of the equation

$$\omega(x, y, -h(x, y, 0), t^*) = 1. \tag{1.3}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 113-120, November-December, 1985. Original article submitted August 29, 1984.

 0021-8944/85/2606-0860$09.50

Equations (1.1)-(1.3) constitute a general model of impact erosion represented as a continuous process, which is not always true. For example, under conditions of high-velocity impact, destroyed (fractured) material is removed in each impact event but cumulative damage is negligible. Such a process can be described phenomenologically, taking $h = h(x, y, t)$ and introducing a coefficient of mass loss per particle [6, 7]. The model in [3] will be valid in a certain intermediate range of impact parameters. In this model, erosion was represented as a discrete-integral process. The model in [3] contains a large number of integral equations and its use poses serious difficulties even in the simplest case [4]. The continuum model is also complicated for the purposes of analysis, since the right side of (1.1) contains random parameters and processes. In fact, each event of particle impact against the surface is a random event. Thus, $\sigma_{ik}(t)$ and $\omega(t)$ are random proesses. The set $\{\xi_\omega\}$ also contains random parameters. It follows from (1.2) that the relief formed on the surface is also described by a random function. Subsequent interaction of a drop with surface irregularities increases the disorder in the system. In general, such a detailed description of erosion as is given by Eqs. (1.1)-(1.3) is not necessary. Thus, it is possible to construct a simpler model which ignores the subtle processes which take place in the system. For example, in [8] a diffusion model of erosion was developed on the basis of the familiar analogy between the processes of thermal and erosive fracture:

$$\partial\omega/\partial t + U\partial\omega/\partial z - D\partial^2\omega/\partial z^2 = \langle\Phi\rangle.$$

Here, $U(t)$ is the rate of erosion; $D(t) = d/dt\langle(h-\langle h\rangle)^2/2)\rangle$. The diffusional model is simple mathematically but does not permit a sufficiently detailed examination of the erosion process. This model is useful in analyzing the beginning of fracture, when $D(t) \approx$ const. However, at a later stage, when $D(t) \to 0$, it is necessary to employ hypotheses regarding the behavior of the function $D(t)$. This causes serious problems if it is necessary to predict parameters of the induced roughness as accurately as possible (such as in modeling turbulence, in calculating reflectivity, etc.). Presented below is one approach to the solution of this problem.

2. Model of Nonsteady Erosion. We will assume that at $t = 0$ a flat specimen is placed in a uniform flow of drops. After the elapse of the latent period, fractured material begins to be carried off. Experiments [3] show that erosion is not seen immediately over the entire surface. Instead, it spreads gradually from the weakest parts of the surface to the move durable elements. We will write the solution of Eq. (1.3) in the form $t^* = t^*(x, y)$. The lifetime interval from t^* to $t^* + \Delta t^*$ corresponds to a certain area on the specimen surface. We will assume that we know the statistic of lifetimes of surface elements $n(t^*)$. Then the sought area is $\Delta s(t^*) = n(t^*)\Delta t^*$. The specimen can be represented as consisting of cells of the area $\Delta s(t^*)$ and as having a total area equal to unity:

$$\int_{t_0^*}^{\infty} n(t^*)\,dt^* = 1 \tag{2.1}$$

(t_0^* is the minimum lifetime).

We will formulate a problem on finding erosion parameters, including surface roughness, for a known statistic $n(t^*)$. The analysis is simplified if we ignore the dependence of the damage accumulation on ω. This limitation is not fundamentally important for an erosion problem but is important in modeling a stress field The scatter of the parameters $\{\xi_\omega\}$ is taken into account by the relation $n(t^*)$, so we set $\Phi(\omega, \sigma_{ik}, \{\xi_\omega\}) = \varphi(\sigma_{ik})$. In Eq. (1.1) we average over the ensemble of particles falling on the given area $\Delta s(t^*)$ during the time Δt. Ignoring the effects of multiple impacts — which is valid in the case of a low volumetric content of particles, we represent the mean sum of the loads in the form

$$\overline{\varphi}(z) = n_p u_p \int_0^{\infty} dt' \int_0^{\infty} 2\pi r\,dr\,\varphi\left(\sigma_{ik}^0(r, z, t')\right), \tag{2.2}$$

where σ_{ik}^0 is the stress tensor in the impact of a drop against a barrier; n_p and u_p are the numerical density and velocity of flow of the particles; the bar corresponds to averaging over the ensemble of particles.

The solution of the problem of the erosive destruction of an individual cell can be represented in the form

$$\bar{\omega}(z, t) = \begin{cases} t\bar{\varphi}(z), & t \leqslant t^*, \\ t^*\bar{\varphi}(z) + \int\limits_{t^*}^{t} \bar{\varphi}(z + \bar{h}(s))\, ds, & t > t^*, \end{cases} \tag{2.3}$$

where $t^* = 1/\bar{\varphi}(0)$ is the duration of the latent period.

At $t \geq t^*$, on the surface we have the equation $\omega(-\bar{h}(t), t) = 1$. Setting $z = -\bar{h}(t)$ in (2.3), we find

$$t^*\bar{\varphi}(\bar{h}(t)) + \int\limits_{t^*}^{t} \bar{\varphi}(\bar{h}(s) - \bar{h}(t))\, ds = 1 \tag{2.4}$$

Equation (2.4) predicts that the dependence of the depth of wear on time is determined by the law of decay of the fracture stresses with propagation into the depth of the material. This result is a very important difference between the present model and the models in [1, 5]. The latter also take into account the lifetime statistic but do not consider the time dependence of the rate of erosion of an individual cell. The solution of Eq. (2.4) can be represented in the form $\bar{h} = \bar{h}(t, t^*)$. To calculate the mean depth of wear, this expression must be averaged over the ensemble of cells:

$$\langle \bar{h} \rangle (t) = \int\limits_{t_0^*}^{t} \bar{h}(t, t^*)\, n(t^*)\, dt^*. \tag{2.5}$$

The induced roughness can be described by the rms gradient of the levels of the surface relief

$$\delta h^2 = \int\limits_{t_0^*}^{t} (\bar{h}(t, t^*) - \langle h \rangle (t))^2\, n(t^*)\, dt^*. \tag{2.6}$$

Equations (2.2)-(2.6) solve the state problem of finding the parameters of nonsteady erosion from a specified lifetime statistic.

Let us examine two fracture regimes which can be analyzed without having a detailed picture of the stress state. In Eq. (2.4) we set $\bar{\varphi}(s) = e^{-ks}/t^*$, with k being the decay parameter. The corresponding solution describes the regime in which the erosion rate is steady

$$\bar{h}(t) = (t - t^*)/kt^*, \quad t \geqslant t^*. \tag{2.7}$$

It seems intuitively that the rate of erosion of an individual cell should ultimately reach a constant value and that Eq. (2.7) should therefore be satisfied. However, it is easily shown that the rms roughness increases over time in this regime as $\delta h^2 = (t^2/k^2)[\langle (1/t^*)^2 \rangle - \langle 1/t^* \rangle^2]$. The growth of depressions on the surface obviously cannot continue for an infinitely long time. We require that as $t \to \infty$ the condition $\delta h^2 = \text{const}$ be satisfied. This condition corresponds to a fracture regime in which the rate of erosion of an individual cell is self-similar relative to the time of beginning of fracture ($d\bar{h}/dt = 1/kt$). Thus, the depth of wear

$$\bar{h}(t) = k^{-1} \ln (t/t^*). \tag{2.8}$$

In this process $\delta h^2 = k^{-2}(\langle \ln^2 t^* \rangle - \langle \ln t^* \rangle^2)$, while the profile of the fracture stresses has the form

$$\bar{\varphi}(z) = \begin{cases} (1 + kz)/t^*, & -1 < kz \leqslant 0, \\ 0, & kz \leqslant -1. \end{cases}$$

It follows from (2.8) that at $t \to \infty$ the mean depth of wear increases as $\langle h \rangle(t) = k^{-1}\ln(t/\tau)$, where $\tau = \exp[\langle \ln t^* \rangle(\infty)]$. Such a law is actually observed in experiments. Figure 1a shows data from [9] on the erosion of 12% chromium steel in a flow of drops. The data was analyzed in the coordinates $k\langle h \rangle(t) - \ln(t/\tau)$, where points 1-5 correspond to the following values of impact velocity: 198; 229; 256; 284; 311 m/sec. It is evident that all of the points are tightly grouped around a line with single slope, i.e., the increase in wear depth obeys a logarithmic law.

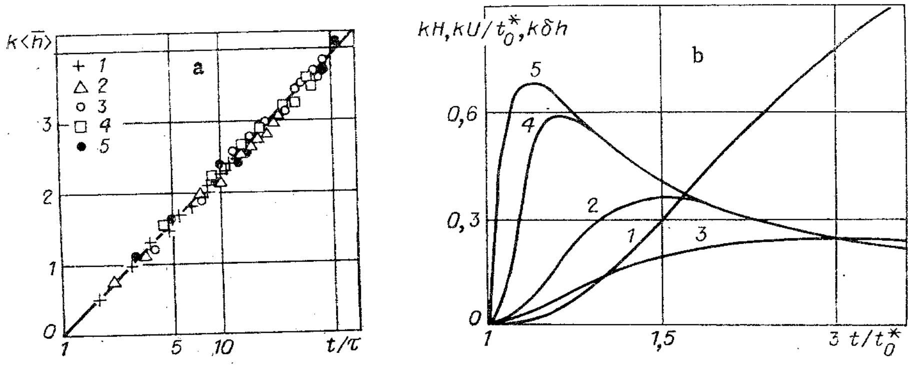

Fig. 1

To find the function $n(t^*)$, we can draw an analogy between the fatigue fracture of a system of cells on an eroded surface and a system of identical elastic elements such as springs, rods, etc. [5]. It was established by analyzing the experimental data in [10] that the Weibull distribution

$$n(t^*) = \lambda\mu\,(t^* - t_0^*)^{\mu-1}\exp\left[-\lambda\,(t^* - t_0^*)^{\mu}\right], \tag{2.9}$$

often used to evaluate reliability in fatigue fracture, is quite applicable to many materials under conditions of erosive fracture as well (in [5], we assumed $\mu = 1$; in reality, the parameter μ changes within a broad range for different materials).

Figure 1b shows time dependences of the depth of wear, rate of erosion, and rms roughness (curves 1-3, respectively) calculated for a logarithmic erosion law (2.8) on the assumption that lifetime distribution law (2.9) is satisfied with $\mu = 2$. There is qualitative agreement between the predicted relations and the observed laws of erosive fracture both under cavitation conditions [2] and under the influence of drops [4-6, 9, 10]. To a certain extent, these results answer the questions raised in [2]. This applies especially to the issue of whether or not a constant erosion rate is established during fracture. It can definitely be said that the erosion rate cannot be constant in a fatigue process such as described above. However, if material is removed with each impact event and fracture is not related to damage accumulation, then a steady erosion rate may be established. For example, in the impact of solid particles, there is almost no incubation period, and the erosion rate reaches a steady value almost immediately. In certain cases there is a delay in fracture during erosion by solid particles as well, but this effect is attributable to the simultaneous occurrence of erosion and the deposition of particles on the surface in the flow [11].

It is evident from Fig. 1b that the erosion rate reaches a maximum value and then decreases. Meanwhile, the maximum erosion rate increases with an increase in λ ($\lambda = 1$ and 8 and $\mu = 2$ for curves 2 and 4, respectively, while $\lambda = 8$ and $\mu = 1$ for curve 5). The maximum erosion rate is expressed through the parameters λ and μ in the form

$$U_m = (1/kt_0^*)\,\mu q^{1/\mu}\,\xi^{1-1/\mu}e^{-\xi}, \tag{2.10}$$

where $\xi(\mu, q)$ is the solution of the equation

$$\xi = \ln\left[1 + \mu\xi\,(1 + (q/\xi)^{1/\mu})\right],\ \xi > 0,\ q = \lambda t_0^{*\mu}.$$

Figure 2 shows the dependence of the maximum erosion rate on the parameter q for $\mu = 1$, 2, and 3 (curves 1-3, respectively). These results show that the dispersion of the strength properties of the material is an important parameter characterizing resistance to erosion. Given the same minimum lifetime of the material, resistance to erosion decreases when λ increases.

It is evident from Eq. (2.10) and Fig. 2 that the maximum erosion rate depends in a complex manner on the duration of the latent period, i.e. [5] notwithstanding, there is a single relation $U_m(t_0^*)$. This is apparently the reason for such appreciable scatter of data when analyzed in the coordinates U_m–t_0^* (this scatter is one order of magnitude and more in [5]). Thus, the maximum erosion rate is not a very good parameter for comparing the

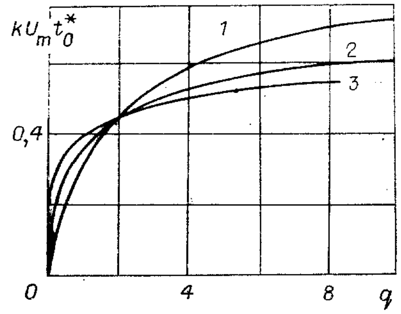

Fig. 2

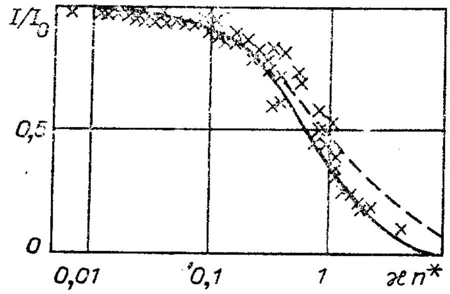

Fig. 4

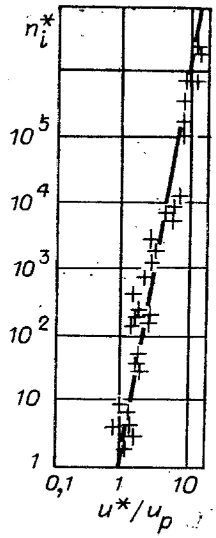

Fig. 3

erosion resistance of different mateials. It follows from the data in Fig. 1 that the decay parameter k and the effective time of erosion $\tau = \exp[\langle \ln t^* \rangle(\infty)]$ can be used as a characteristic of the capacity of a material to resistance erosion. Thus, in the general case, the time dependence of erosion parameters can be described by two dynamic (k, t_0^*) and two statistical (λ, μ) parameters.

3. Patent Period of Erosion. To simplify the analysis, we will assume that the material retains its elastic properties right up to fracture and that the following impulsive fracture criterion [12] is satisfied

$$\varphi(\sigma_{ik}) = \begin{cases} \nu_0(\sigma/\sigma_* - 1), & \sigma > \sigma_*, \\ 0, & \sigma \leqslant \sigma_*, \end{cases} \qquad (3.1)$$

where σ is the maximum normal tensile stress; σ_*, threshold value of the breaking stresses; ν_0, characteristic rate of damage accumulation.

The problem of the propagation of stress waves in an elastic half-space for loading conditions simulating the impact of a drop against a barrier was examined in [13, 14]. The true distribution of the normal load was replaced by the mean pressure applied to the contact spot, the radius of which changed in accordance with the law $r_c(t) = \sqrt{d_p u_p t}$. Thus, it is possible to calculate the stresses at the stage of compression of the cutoff signal. It was established in [14, 15] by comparison of calculated stresses with the observed fracture pattern that Rayleigh waves play the main role in destruction of the surface. According to [15], the radius of the fracture zone may be one order greater than the size of the central undamaged spot. For these conditions, the peripheral region makes the main contribution to the integral (2.2). Thus, the damages can be evaluated by using the stress values in the Lamb problem with an equivalent concentrated load [12]:

$$\sigma_R = (p_c u_p d_p / 2c_s^2 t)\, \sigma_R^0(\xi,\ 0,\ \gamma), \qquad (3.2)$$
$$\sigma_R^0 = [(2-\theta^2)/2\theta c_0]\left[\frac{2(1-\gamma^2)}{\sqrt{\theta^2-\xi^2}} - \frac{1}{\theta + \sqrt{\theta^2-\xi^2}}\right],$$

where p_c is the mean impact pressure; $\xi = r/c_s t$; $\theta = c_R/c_s$; $\gamma = c_t/c_s$; $c_{s,t,R}$ is the velocity of the longitudinal, transverse, and surface waves, respectively;

$$c_0 = \frac{\sqrt{1-\gamma^2\theta^2}}{\sqrt{1-\theta^2}} + \gamma^2 \frac{\sqrt{1-\theta^2}}{\sqrt{1-\gamma^2\theta^2}} + \theta - 2.$$

Inserting (3.2) into (3.1) and integrating, we obtain

$$d\omega/dt\,|_{z=0} = n_p u_p \pi R_*^3 \nu_0 F(\gamma)/c_R, \tag{3.3}$$

where $R_* = p_c u_p d_p / 4\sigma_* c_R$ is the characteristic dimension of the fracture zone; $F(\gamma)$ is a numerical coefficient dependent on the Poisson's ratio. The duration of the incubation period is determined from the equation $\omega(t_0^*) = 1$. Integrating (3.3), we find $t_0^* = c_R/[\pi R_*^3 \nu_0 n_p u_p F(\gamma)]$. The product $n_i^* = t_0^* n_p u_p \pi d_p^2/4$ is the incubation period after Sprinzher [5]. Choosing $p_c = 1.5\rho_l c_l u_p/(1 + \rho_l c_l/\rho_s c_s)$ to express the dependence of impact pressure on the impact parameters [4], we have

$$n_i^* = 4.74\sigma_*^3 c_R^4 (1 + \rho_l c_l/\rho_s c_s)^3/[\nu_0 d_p \rho_l^3 c_l^3 u_p^6 F(\gamma)], \tag{3.4}$$

where ρ_ℓ, c_ℓ are the density and speed of sound in the liquid; ρ_s is the density of the material of the barrier.

Equation (3.4) is interesting in the sense that the dependence of n_i^* on impact velocity agrees qualitatively with the relation presented in [5]. Figure 3 shows different results generalized in [5] together with Eq. (3.4) (solid line) in the form $n_i^* = (u^*/u_p)^6$. Sprinzher, performing a similar analysis, used the quasistatic part of the tensile stresses $\sigma_{ro}^0 = \gamma^2/\xi^2(1 - \gamma^2)$ in his calculations and obtained the relation $n_i^* \sim u_p^{-1}$. According to [5], the data in Fig. 3 is generalized by the relation $n_i^* \sim u_p^{-5.7}$. We could point out the difficulty connected with determining t_0^*, but there is nevertheless good agreement between the theoretical (3.4) and experimental (Fig. 3) values. The effect of drop diameter on the duration of the incubation period was studied in [9]. It was found that the product $t_0^* d_p^3$ remains constant with a change in d_p from 350 to 1050 μm. This means that $n_i^* \sim d_p^{-1}$. This is consistent with Eq. (3.4) but contradicts [5], where it was assumed that the quantity n_i^* is independent of the particle size.

If an optically transparent material is subjected to erosion, then its transparency decreases with time due to scattering of light on microcracks. The decrease in intensity as light propagates into the depth of the material is described by the equation

$$dI/dz = \sigma_f n_f I, \quad z < 0, \tag{3.5}$$

where σ_f is the scattering cross section on a single crack; n_f is the number of cracks in a unit volume.

The scattering cross section for visible light is proportional to the center section of the microcrack ($\sigma_f \sim v_f^{2/3}$, v_f is the volume of the microcrack). Since $\omega = n_f v_f$, it is necessary to formulate a law of change of one of the multipliers. We adopt the following damage model: in each impact event per unit volume of the medium n_{f0} cracks of volume $v_f = v_f(\sigma)$ are created. For subsequent load cycles, v_f = const, $n_f = n^* n_{f0}$, $n^* = n_p u_p t \pi d_p^2/4$ is the number of load cycles. This means that $\sigma_f n_f \sim \omega/v_f^{1/3} = \omega^{2/3}(n_{f0} n^*)^{1/3}$. From here, integrating (3.5), we obtain

$$I/I_0 = \exp(-\varkappa n^*), \tag{3.6}$$

where $\varkappa = a n_{f0}^{1/3}/k n_i^{*2/3}$, and a is a numerical coefficient. This model is valid in the initial stage of the process (before the microcracks begin to merge). Equation (3.6) can be compared with the results in [5]: $I/I_0 = 1/(1 + 10^{-4}\Omega^4 n^*)$, $\Omega = (p_c\sqrt{L_0}/k_c)[1 + 2.19(1 - 2\nu)]$, L_0 is the initial size of the microcracks and k_c is the critical stress intensity factor (actually, the quantity L_0 was used in [5] as a refining parameter). When $\Omega^4 n^* \sim 1$, we have $I_0/I - 1 \approx 10^{-4}\Omega^4 n^* \sim u_p^5$. On the other hand, from (3.6) at $\kappa n^* \ll 1$ we find $I_0/I - 1 \approx \varkappa n^* \sim u_p^5$, i.e., Eq. (3.6) correctly reflects the effect of impact velocity.

Figure 4 shows Eq. (3.6) (solid line) together with different results generalized in [5] by the relation shown by the dashed line. It is evident that Eq. (3.6) agrees better with the experimental results at $\kappa n^* \gtrsim 1$ than does the relation in [5].

The qualitative relations obtained above are also valid in the case of the more general fracture criterion [16] $\dot{\omega} = v_0 (\sigma/\sigma_* - 1)^m \ (m > 0)$. To obtain quantitative results, it is necessary to use a more realistic model of the interaction of a drop with a barrier and to discuss the assumption of the elastic character of deformation of the barrier.

LITERATURE CITED

1. F. G. Heymann, "On the time dependence of the rate of erosion due to impingement or cavitation," in: Erosion by Cavitation or Impingement, Am. Soc. Test. Mater. Spec. Tech. Publ., No. 408 (1967).
2. Plesset and Devane, "Time dependence of cavitational fracture," Trans. ASME Ser. D, No. 4 (1966).
3. W. F. Adler, "Analysis of particulate erosion," Wear, 37, 345 (1976).
4. W. F. Adler, "Mechanics of the impact of liquids," in: Erosion [Russian translation], Mir, Moscow (1982).
5. J. S. Sprinzher, Erosion by Drops [in Russian], Mashinostroenie, Moscow (1981).
6. A. P. Trunev and V. M. Fomin, "Flow of a two-phase flow of the gas—solid-particle type about bodies with allowance for erosion," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1983).
7. A. P. Trunev and V. M. Fomin, "Erosion of a blunt body in a dust-bearing hypersonic flow," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1984).
8. A. P. Trunev and V. M. Fomin, "Phenomenological model of fraction of materials in a two-phase flow," in: Numerical Methods of Solving Problems of the Theory of Elasticity and Plasticity [in Russian], Novosibirsk (1984).
9. D. W. C. Baker, K. H. Jolliffe, and D. Pearson, "The resistance of materials to impact erosion damage," Philos. Trans. R. Soc. London Ser. A, 260, 193 (1966).
10. H. Bush, G. Hoff, and G. Langbein, "Rain erosion properties of materials," Philos. Trans. R. Soc. London Ser. A, 260, 168 (1966).
11. J. H. Neilson and A. Gilchrist, "Erosion by a stream of solid particles," Wear, No. 11 (1968).
12. V. S. Nikiforovskii and E. I. Shemyakin, Dynamic Fracture of Solids [in Russian], Nauka, Novosibirsk (1974).
13. R. M. Blowers, "On the response of an elastic solid to droplet impact," J. Inst. Math. Its Appl., 5, No. 2 (1969).
14. A. G. Evans and T. R. Wilshaw, "Dynamic solid particle damage in brittle materials: an appraisal," J. Mater. Sci., 12, No. 1 (1977).
15. F. R. Tuler and V. M. Butcher, "A criterion for the time dependence of dynamic fracture," Int. J. Fract. Mech., 4, No. 4 (1968).